## Vortex core shrinkage in a two gap superconductor: application to MgB<sub>2</sub>

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## Abstract

As a model for the vortex core in MgB<sub>2</sub> we study a two band model with a clean  $\sigma$  band and a dirty  $\pi$  band. We present calculations of the vortex core size in both bands as a function of temperature and show that there exists a Kramer-Pesch effect in both bands even though only one of the bands is in the clean limit. We present calculations for different  $\pi$  band diffusivities and coherence lengths.

Key words: Magnesium diboride, vortex core, Kramer-Pesch effect

PACS: 74.20.-z, 74.25.Op, 74.70.Ad

In a recent work [1] we have studied the vortex core shrinkage at low temperatures (Kramer-Pesch effect [2]) in a two-gap superconductor. In a clean single gap superconductor the Kramer-Pesch effect results in a linear decrease of the vortex core size as a function of temperature [2]. However, in the dirty limit the size of the vortex core saturates to a finite value of the order of the coherence length and the Kramer-Pesch effect is absent. In the two gap superconductor MgB<sub>2</sub> we have an interesting new situation: in high quality MgB<sub>2</sub> samples it is believed that the  $\sigma$  bands are in the clean, while the  $\pi$  bands are still in the dirty limit [3,4]. This motivated us to study the Kramer-Pesch effect within a mixed model, which was proposed recently by Tanaka et al [5], consisting of a clean  $\sigma$  band and a dirty  $\pi$ band. The surprising result of our study was that the clean band induces a Kramer-Pesch effect in the dirty band [1], i.e. the vortex core size decreases linearly as a function of temperature even in the dirty  $\pi$  band. This "induced" Kramer-Pesch effect should be very interesting for experimental verifications of

In the present work we want to study the influence of the  $\pi$  band coherence length on the induced Kramer-Pesch effect. The coherence length in the clean  $\sigma$  band is given by  $\xi^{(\sigma)} = \hbar v_{F,\sigma}/\Delta^{(\sigma)}$  and thus does not vary much with the amount of impurities in the  $\pi$  band. However, the coherence length in the dirty  $\pi$  band is given by  $\xi^{(\pi)} = \sqrt{\mathcal{D}^{(\pi)}/2\pi T_c}$ , where  $\mathcal{D}^{(\pi)}$  is the diffusivity in the  $\pi$  band. Thus, the  $\pi$  band coherence length varies with the impurity scattering rate and should be sample dependent.

For our numerical calculations we use the Riccati method to solve Eilenberger's equations in the clean band, while for the dirty band we solve Usadel's equations in the vicinity of a single vortex. The gaps in both bands are calculated self-consistently and the vortex core radii are calculated from the slope of the gap function at the vortex center as has been suggested by Hayashi et al [7]:

the Kramer-Pesch effect. Experimentally in MgB<sub>2</sub> it is much easier to image the vortex core in the  $\pi$  band than in the  $\sigma$  band by scanning tunneling microscopy (STM), because tunneling is easier into the  $\pi$  band [6].

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$$\left(\xi_V^{(\alpha)}\right)^{-1} = \left. \frac{\partial \Delta^{(\alpha)}(r)}{\partial r} \right|_{r=0} \frac{1}{\Delta^{(\alpha)}(r=\infty, T)} \tag{1}$$

Parameters appropriate for MgB<sub>2</sub> have been used as has been described in our previous work [1].

In Fig. 1 we show the temperature dependence of the vortex core radius in both bands relative to the (fixed)  $\sigma$  band coherence length for three different values of the  $\pi$  band coherence length. The vortex core radius in the  $\sigma$  band  $\xi_V^{(\sigma)}$  is only weakly affected by the change of the  $\pi$  band coherence length and shows a clear linear decrease, similar as in a clean single band superconductor. However, the vortex core radius in the  $\pi$  band  $\xi_V^{(\pi)}$  strongly depends on the  $\pi$  band coherence length and its temperature dependence shows some deviations from the linear behavior. This can be seen more clearly in the insets, where we are showing the temperature dependence of the ratio of the vortex core sizes in the two bands  $\xi_V^{(\pi)}/\xi_V^{(\sigma)}$ . For a large  $\pi$  band coherence length (lowest panel) this ratio increases almost linearly with decreasing temperature, while for a short  $\pi$  band coherence length (upper panel) the  $\pi$  band vortex core size at high temperature mostly follows the  $\sigma$  band vortex core size and then decreases more slowly at low temperatures. However, it is very clear that in all three cases the  $\pi$  band vortex core size eventually goes to 0 for  $T \to 0$ , even though the  $\pi$  band is in the dirty limit. Note, that the  $\pi$  band vortex core is always larger than the vortex core in the  $\sigma$  band, even in the case where the  $\pi$  band coherence length is much smaller than the  $\sigma$  band coherence length. We attribute this to the fact that the dirty  $\pi$  band cannot support its own vortex core shrinkage and is mostly induced by the one in the  $\sigma$  band.

We conclude that the induced Kramer-Pesch effect reported earlier [1] exists independently of the diffusivity and coherence length in the dirty band.

## References

- A. Gumann, S. Graser, T. Dahm, and N. Schopohl, Phys. Rev. B 73, 104506 (2006).
- [2] L. Kramer and W. Pesch, Z. Physik **269**, 59-64 (1974).
- [3] J. W. Quilty, S. Lee, S. Tajima, and A. Yamanaka, Phys. Rev. Lett. 90, 207006 (2003).
- [4] I. Pallecchi, V. Braccini, E. G. d'Agliano, M. Monni, A. S. Siri, P. Manfrinetti, A. Palenzona, and M. Putti, Phys. Rev. B 71, 104519 (2005).

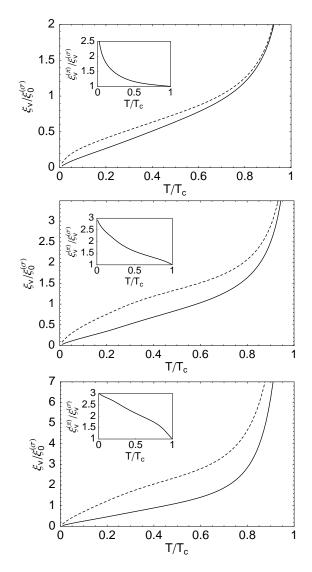


Fig. 1. Temperature dependence of the vortex core radius  $\xi_V/\xi^{(\sigma)}$  in the  $\sigma$  band (solid line) and in the  $\pi$  band (dashed line) for different values of the coherence length ratio  $\xi^{(\sigma)}/\xi^{(\pi)}$ . (a)  $\xi^{(\sigma)}/\xi^{(\pi)}=5$ , (b)  $\xi^{(\sigma)}/\xi^{(\pi)}=1$ , and (c)  $\xi^{(\sigma)}/\xi^{(\pi)}=0.2$ . The insets show the temperature dependence of the ratio of the two vortex core radii  $\xi_V^{(\pi)}/\xi_V^{(\sigma)}$ .

- [5] K. Tanaka, D. F. Agterberg, J. Kopu and M. Eschrig, cond-mat/0512118.
- [6] M. R. Eskildsen et al, Phys. Rev. Lett. 89, 187003 (2002).
- [7] N. Hayashi, Y. Kato and M. Sigrist, J. Low Temp. Phys. 139, 79 (2005)